

# 1.3

## Focus on Connecting

Situations in real life often involve problems that you can solve using mathematical processes. You can make connecting mathematics to real life easier by drawing a diagram to illustrate the situation. You can then use the diagram to help you solve the problem. You can also use other strategies, such as making an organized list or looking for a pattern.

Often, in solving problems, you need to make connections among different areas of mathematics. For example, to find the amount of paint needed, you would use measurement, geometry, and number skills.



### Investigate

#### How can you connect different representations of a problem?

A Superball is dropped from a height of 160 cm. When it lands on the ground, it bounces to a height that is 75% of the previous height. What is the total distance travelled by the ball at the moment it bounces for the fifth time?

#### Understand the Problem

#### Choose a Strategy

#### Carry Out the Strategy

#### Reflect

1. Read the problem above. Read it again. Express the problem in your own words.
2. A good strategy is to draw a diagram to illustrate the bouncing of the ball. What will your diagram look like? What labels will you put on your diagram? What calculations will you need to do?
3. Draw a diagram representing the path of the bouncing ball. Label the diagram with the appropriate measurements. Do you need to show all five bounces? Why?
4. Does your answer seem reasonable? If you are not sure, what should you do?

## Example 1 Number of Point Totals

The Aces hockey team has played five games. A win is worth 2 points, a tie is worth 1 point, and a loss is worth 0 points. Determine the number of different point totals the Aces could have after five games.

### Solution

Make an organized list to help with the solution.

Wins	Losses	Ties	Total Points
5	0	0	10
4	1	0	8
4	0	1	9
3	2	0	6
3	1	1	7
3	0	2	8
2	3	0	4
2	2	1	5
2	1	2	6
2	0	3	7
1	4	0	2
1	3	1	3
1	2	2	3
1	1	3	5
1	0	4	6
0	5	0	0
0	4	1	1
0	3	2	2
0	2	3	3
0	1	4	4
0	0	5	5

Each win is worth 2 points.  $5 \times 2 = 10$ .

There are 11 possible point totals—between 0 and 10 points.

*How many hockey pucks are there in Canada?*

Many problems, like this one, do not have exact answers.

A **Fermi problem** is one that uses estimation in its solution. It is solved by asking appropriate questions, whose answers lead to the next stage in the solution. Making connections and using various mathematical skills are important aspects of the solution.

### Did You Know?

Fermi problems are named after Enrico Fermi (1901–1954). He was a well-respected Italian physicist who liked to pose these problems. See Example 2 on the next page.

## Example 2 Jelly Beans in a Jar

How many jelly beans will fill a 4-L jar?



### Solution

#### Understand the Problem

I need to figure out the size of a jelly bean, then how many fit into 4 L. I will need to account for space between the beans.

#### Choose a Strategy

Use estimation and make assumptions.

What shape is a jelly bean?

It is roughly the shape of a cylinder.

It is about 1.5 cm long and about 0.5 cm in radius.

I need to make some assumptions about the shape and size of a jelly bean.

#### Carry Out the Strategy

Find the approximate volume of a jelly bean.

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi (0.5)^2 (1.5) \\ &\doteq 1.2 \end{aligned}$$

I need to use the formula for the volume of a cylinder.

The volume of a jelly bean is about  $1.2 \text{ cm}^3$ .

What is the volume of the jar in cubic centimetres?

$$1 \text{ L} = 1000 \text{ cm}^3$$

$$4 \text{ L} = 4000 \text{ cm}^3$$

Adjust for the space between the jelly beans.

Air might take up about 10% of the volume.

So, only 90% of the jar's volume will be jelly beans.

10% is a reasonable guess for the amount of air.

$$\begin{aligned} 90\% \text{ of } 4000 &= 0.9 \times 4000 \\ &= 3600 \end{aligned}$$

Now, I need to use my skills with percent.

$$\begin{aligned} \text{Number of jelly beans} &= \text{volume} \div \text{volume of a jelly bean} \\ &= 3600 \div 1.2 \\ &= 3000 \end{aligned}$$

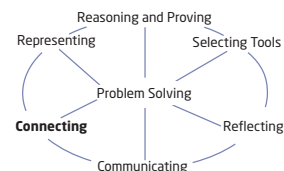
About 3000 jelly beans will fill a 4-L jar.

#### Reflect

A 4-L jar is pretty large. I could test the answer by seeing how many jelly beans fit into a 250-mL cup, then multiplying the count by 16 (because  $16 \times 250 \text{ mL} = 4000 \text{ mL}$  or 4 L).

## Key Concepts

- You can make connections that relate math to other areas of study and of daily life.
- You can also make connections between areas of mathematics, such as geometry and number sense.



## Communicate Your Understanding

- C1** Using the Investigate as a reference, explain how drawing a diagram is different from drawing a picture.
- C2** How many people are sitting down in your school at this moment? What information will you need to find or estimate to solve this Fermi problem?

## Practise

For help with question 1, see the Investigate.

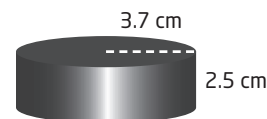
1. A bird flew 800 m in one direction. It turned around and flew half as far back. Then, it turned around and again flew half as far back. The bird continued this pattern for a total of six flights.
  - a) Draw a diagram illustrating this situation.
  - b) What was the total distance the bird flew?
  - c) How far from its starting point did the bird end up?

For help with question 2, see Example 1.

2. Raoul has four Canadian coins in his pocket. The coins are quarters, dimes, or nickels. What are the possible total values of the coins?

For help with questions 3 and 4, see Example 2.

3. How many hockey pucks would fit inside your classroom? Explain your reasoning.
4. The tires on Honi's bike are guaranteed to last 2000 km. She uses her bike mostly to ride to school, which is 8 km from her home. How many years can she expect the tires to last? Explain your reasoning.

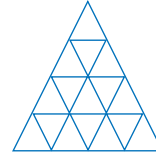


## Connect and Apply

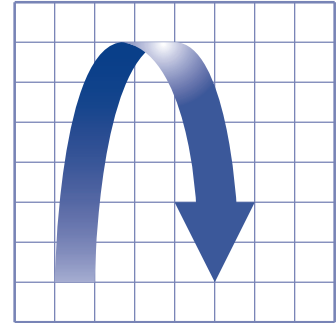
In each problem, write a sentence to describe your strategy. Then, carry out your strategy.

5. Five friends ate a 12-slice pizza. Samir ate three times as many slices as Joe. Joe ate half as many slices as Emily. Kendra and Fong together ate half a pizza. Kendra ate one third as many slices as Samir. Fong ate the most slices. What fraction of a pizza did each person eat?

6. How many triangles of all sizes are there in the diagram?



7. If each square on the grid measures 0.5 cm by 0.5 cm, what is the approximate area of the arrow?



8. A snail begins climbing a pipe from a point 20 m below the ground. Each day, the snail climbs 4 m and slides back 3 m. How long will it take to reach the top of the pipe, which is 7 m above the ground?
9. How many times does a cat's heart beat in a lifetime?



### Extend

10. Each three by three square, each row, and each column must contain each of the numerals 1 through 9 only once. Copy and complete this Sudoku puzzle.

	6		7		3			4
2		7	6		4		9	
4		5		2		6		
1	7			8				
			9		1			
				3			6	8
		3		4		8		6
	1		5		6	3		9
6			3		8		2	

11. Design your own geometry problem in which a diagram would be useful to help solve the problem.
12. A polygon has 20 sides. How many diagonals does it have?